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Effects of magnetic impurities in the *t*–*J* model

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Abstract. We study the effects of magnetic impurities in the strongly correlated electron system from the open *t*–*J* model where the two impurities are coupled to the electron system. The kinetic energy of our model is nonlinear in the momentum of the electrons and it is integratable. The interaction parameters can be changed from the ferromagnetic case to the antiferromagnetic case. We obtain the thermodynamic Bethe ansatz equations for the excitation state. The effects of the magnetic impurities on magnetization and charge fluctuation are studied. The finite-size correction of the free energy due to impurities is obtained and some special limits of the system are discussed. We find that the system has boundary bound states introduced by the impurities and formed by four imaginary modes of the rapidities.

1. Introduction

The impurity models have attracted considerable interest in condensed matter physics since magnetic Ni and nonmagnetic Zn impurity effects provide a simple way to understand the role of electrons on the Cu–O plane and in one dimension even the smallest amount of defects may drastically change the properties of the electron system. In the Kondo and Anderson impurity models [1, 2], magnetic impurities are embedded in noninteracting metals. The magnetic impurity is coupled to the Heisenberg spin chain by Andrei and Johannesson [3] with an arbitrary spin, and extended to the Babujian–Takhtajan spin chain by Lee and Schlottmann [4,5]. The Kondo problem is devoted to studying the effect due to exchange interaction between the impurity spin and electron gas and in its original treatment the electron–electron interaction is discarded. This is reasonable in three dimensions where the interacting electron system can be described by a Fermi liquid. The recent advances in semiconductor technology enable one to fabricate very narrow quantum wires which can be considered as one-dimensional and furnish a real system of a Luttinger liquid. Edge states in a two-dimensional electron gas for the fractional quantum Hall effect can also be considered as a Luttinger liquid [6]. Intense effort and much progress has been made around the subjects using different approaches such as renormalization techniques [7, 8], bosonization, boundary conformal field theory [9] and non-Fermi liquid theory [10–12] .

The theory of the magnetic impurities in the Fermi liquid and Luttinger liquid [13, 14] has been recently studied where the few impurities are coupled with strongly correlated electron

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systems [15]. The underscreened Kondo effect is studied in the magnetic impurity model by Karyn Le Hur and Coqblin [16]. In [17], the local magnetic moments around an impurity site are considered. The exact solutions of the integrable models on the subjects are useful [18–20], from which one can expect to draw definite conclusions. Indeed, Schlottmann and Zvyagin introduced the impurity in the supersymmetric t–J model via its scattering matrix with the itinerant electrons [21, 22]. Then, in principle, the Hamiltonian of the system and other conserved currents can be constructed by the transfer matrix. They discussed the magnetic impurities embedded in the Hubbard model [23]. Bedüfig *et al* solved the integrable model with the impurity coupled with a periodic t–J chain [24], in which the impurity is introduced through local vertices. By taking into account the backward scattering, we discuss the effects of the magnetic impurities in a strongly correlated electron system by using the open boundary system [25–27] with the impurities at the open ends of the model. Based on Kane and Fisher's observation [8], we see this is advantageous and this programme has been used in [28–32].

The t–J model is one of the most fundamental systems of strongly correlated electrons for providing understanding of the behaviour of the electrons for high-T_c superconductivity [33, 34]. It describes the nearest-neighbour hopping of electrons with spin-exchange interaction. By considering the very strong repulsive on-site Coulomb interaction, the doubly occupied lattice sites are prohibited for the electrons so that there are only three possible states at each lattice site for $\frac{1}{2}$ spin. Recently, the theory of magnetic impurities in Fermi and Luttinger liquids [13, 14] has been focused on the properties of the Luttinger liquid in the t–J model [35–37] as discussed in [38]. Recently, Zvyagin found that in the case when the impurity is connected to the host via a 'weak link' its low-energy behaviour coincides with that of an impurity in a periodic chain. Schlottmann and Zvyagin consider a finite concentration of magnetic impurities embedded in a one-dimensional lattice via scattering matrices [39]. Very recently, the impurity model related to the t–J model was also studied in [40], and the hidden Kondo effect in the correlated electron chain was investigated [41].

In [29], we studied the magnetic impurities with the $\frac{1}{2}$ spins in the framework of the open $SU(3)$ invariant t–J model. The electrons in the triplet states are scattered but those in singlet states are not. Magnetic impurities with arbitrary spins are discussed in [32]. For the bulk the model is isomorphic to the spin-1 Heisenberg chain with $SU(3)$ invariance. It has no graded super-algebra and corresponds to three bosonic degrees of freedom. The integrability of the impurity model related to the traditional t–J model, which has the graded FFB super-algebra, and the one related to $SU(3)$ invariant t–J model are studied in detail in [31].

In this paper we continue the analysis of the exactly solvable impurity model within the framework of the open boundary traditional t–J model where the electrons scatter in singlet states. The Hamiltonian of this system can be written as

$$H = -\mathcal{P}\left\{\sum_{j=1}^{G-1}\sum_{\sigma=\uparrow\downarrow}(C^+_{j\sigma}C_{j+1\sigma} + C^+_{j+1\sigma}C_{j\sigma})\right\}\mathcal{P} \pm 2\sum_{j=1}^{G-1}\boldsymbol{S}_j\cdot\boldsymbol{S}_{j+1} \mp \tfrac{1}{2}\sum_{j=1}^{G-1}n_jn_{j+1}$$
$$+J_a\boldsymbol{S}_1\cdot\boldsymbol{S}_a + V_an_1 + J_b\boldsymbol{S}_G\cdot\boldsymbol{S}_b + V_bn_G \qquad (1)$$

where $C^+_{j\sigma}$ ($C_{j\sigma}$) is the creation (annihilation) operator of the conduction electron with spin σ on the site j; $J_{a,b}$, $V_{a,b}$ are the Kondo coupling constants and the impurity potentials, respectively, and can be parametrized as $J_{a,b} = \mp 8/\{(2C_{a,b} \mp 1)(2C_{a,b} \pm 3)\}$, $V_{a,b} = \pm(3 - 4C^2_{a,b})/\{(2C_{a,b} \mp 1)(2C_{a,b} \pm 3)\}$ with the arbitrary constants $C_{a,b}$; $\boldsymbol{S}_j = \frac{1}{2}\sum_{\sigma,\sigma'}C^+_{j\sigma}\sigma_{\sigma,\sigma'}C_{j\sigma'}$ is the spin operator of the conduction electron; $n_j = C^+_{j\uparrow}C_{j\uparrow} + C^+_{j\downarrow}C_{j\downarrow}$ is the number operator of the conduction electron; G is the length (or site number) of the system. We have set that $J = \pm 2$ and $V = \mp\frac{1}{2}$ in the above Hamiltonian. The projector $\mathcal{P} = \prod_{j=1}^{G}(1 - n_{j\uparrow}n_{j\downarrow})$ restricts the Hilbert space by the constraint of no double occupancy at one lattice point. In the first quantization the above Hamiltonian can be expressed by using translation operators [42].

The arrangement of this paper is as follows. In section 2, the low-field magnetization of the impurity model is derived which means that the susceptibility also has logarithmic correction when the impurities exist. In section 3, the charge fluctuation of the system is discussed for the most important case, that is, the two electrons in the singlet state are scattered but they are not scattered in a triplet state. Some results of the integration limits of the impurity model are also presented. In section 4, the excitation of the system is described based on the thermal Bethe ansatz equations. By minimizing the thermodynamic potential, we obtain the free energy of the system with the magnetic impurities. The low-temperature limit and the high-temperature behaviour of the system are discussed and the boundary bound states are obtained; the explicit expression of the free energy due to magnetic impurities is especially given under the limits of the interaction with the conduction band. Finally, we give the concluding remarks.

2. Magnetization

The impurity model (1) is exactly solvable and the Hamiltonian of the system can be diagonalized [29, 31] by using the Bethe ansatz method. The integrability condition means that the interaction parameters of the magnetic impurities with the conduction electrons have special forms (for details see [31]). The distribution density functions $\sigma(k)$ and $\rho(k)$ satisfy the integral equations

$$\begin{aligned} a(k,1) + \frac{1}{2G}\sigma^G(k) &= \sigma(k) + \sigma^h(k) + [2]\sigma(k) + [1]\rho(k) \\ a(k,\frac{1}{2}) + \frac{1}{2G}\rho^G(k) &= \rho(k) + \rho^h(k) + [1]\sigma(k) \end{aligned} \tag{2}$$

for the ground state, where $\sigma^h(k)$ and $\rho^h(k)$ are the hole distribution functions (also see section 4), and $a(\lambda, \eta) \equiv \pi^{-1}\eta/(\lambda^2 + \eta^2)$. The functions $\sigma^G(k)$ and $\rho^G(k)$ are denoted by

$$\begin{aligned} &\sigma^G(k) = a(k, C_a + \tfrac{3}{2}) + a(k, C_a - \tfrac{1}{2}) + a(k, C_b + \tfrac{3}{2}) + a(k, C_b - \tfrac{1}{2}) - a(k, \tfrac{1}{2}) \\ &\rho^G(k) = a(k, C_a + 1) + a(k, C_b + 1) \end{aligned} \tag{3}$$

with the operator $[n]$ defined as [43] $[n]f(k) = \int_{-\infty}^{\infty} a(k-k', \frac{n}{2}) f(k')\,\mathrm{d}k'$ for $n \neq 0$, and $[0]f(k) \equiv f(k)$. Now we study the contributions of the magnetic impurities for the open boundary conditions. We perform the calculations of the dependence of the magnetization and valence of the impurity as functions of the external field, band filling and temperature along the lines of [35,43] for the pure systems and [21] for the periodic cases. The finite-size corrections for the ground state energies are studied in detail in [31]. We now consider the susceptibility of the system, for convenience, only in the nonmagnetic case. When $\chi^s = V/2 - 3J/8 = -1$, (notice that this quantity was interchanged in [35] but corrected in [15]) the magnetization vanishes in the absence of an external field. We can apply an arbitrarily small magnetic field so that the integration limit B is made much larger than any given Q. By the use of Fourier transformations of the Bethe ansatz equations for the ground state we can obtain the integral equations in this case [15, 35]. By taking into account $B \gg Q$, due to the small magnetic field, we can solve the integral equations by iteration. In this way, we obtain the low-field magnetization contributed by the magnetic impurities:

$$S_z^f = -\frac{\mathrm{e}^{-\pi B}}{(2\pi \mathrm{e})^{\frac{1}{2}}}\Delta L \left(1 + \frac{1}{4\pi B} + \frac{\ln 2B}{(4\pi B)^2} + \cdots\right) \tag{4}$$

where

$$\begin{aligned} \Delta L = {} & \mathrm{sign}(C_a + \tfrac{3}{2})\mathrm{e}^{-|C_a+\frac{3}{2}|\pi} + \mathrm{sign}(C_b + \tfrac{3}{2})\mathrm{e}^{-|C_b+\frac{3}{2}|\pi} - \mathrm{e}^{-\frac{\pi}{2}} \\ & + \mathrm{sign}(C_a - \tfrac{1}{2})\mathrm{e}^{-|C_a-\frac{1}{2}|\pi} + + \mathrm{sign}(C_b - \tfrac{1}{2})\mathrm{e}^{-|C_b-\frac{1}{2}|\pi}. \end{aligned} \tag{5}$$

The symbol function sign is defined as $\operatorname{sign} A = 1$ for $A > 0$, 0 for $A = 0$, and -1 for $A < 0$. We call S_z^f the finite-size correction of the low-field magnetization in comparison with the pure system with periodic case. By considering that H is proportional to $\exp(-\pi B)$ the susceptibility also has logarithmic correction when the impurities exist, which is similar to the Heisenberg antiferromagnet [44] and the t–J model [35].

3. Charge fluctuations with $\chi^s = -1$

For the ground state the magnetization vanishes in the absence of an external magnetic field when $\chi^s = -1$. When the band is almost half-filled or almost empty, the solution of the integral equation of the system has a simple form, which we now discuss. When the band is almost half-filled Q is very small. Then the integral equation can be written as

$$\sigma^h(\xi) + \sigma(\xi) - \int_{-Q}^{Q} \sigma^h(\xi')G_1(\xi - \xi')\,\mathrm{d}\xi' = G_1(\xi) + \frac{1}{2G} R^G(\xi) \tag{6}$$

where $G_1(\xi)$ is denoted by the expression $G_1(\xi) = \int_{-\infty}^{\infty} \mathrm{d}\omega\, \mathrm{e}^{-\mathrm{i}\omega\xi} \mathrm{e}^{-\frac{1}{2}|\omega|}/(4\pi \cosh \frac{\omega}{2})$, and $R^G(\xi) = (2\pi)^{-1} \int_{-\infty}^{\infty} \mathrm{d}\omega\, \mathrm{e}^{-\mathrm{i}\omega\xi} \tilde{\sigma}^G(\omega)/(1 + \tilde{a}(\omega, 1))$ with the wave denoting the Fourier transform. This equation can also be solved by iteration. To first order in Q, the distribution function has the form

$$\sigma(\xi) = \left(1 + \frac{2Q\ln 2}{\pi}\right) G_1(\xi) + \frac{1}{2G}\left\{R^G(\xi) + \frac{Q}{\pi} G_1(\xi) \int_{-\infty}^{\infty} \mathrm{d}\omega\, \frac{\tilde{\sigma}^G(\omega)}{1 + \tilde{a}(\omega, 1)}\right\} \tag{7}$$

when $|\xi| > Q$. And $\sigma(\xi) = 0$ when $|\xi| < Q$ where

$$\begin{aligned} R^G(\xi) = {} & \operatorname{sign}(2C_a + 3)G_{|2C_a+3|-1}(\xi) + \operatorname{sign}(2C_b + 3)G_{|2C_b+3|-1}(\xi) \\ & + \operatorname{sign}(2C_a - 1)G_{|2C_a-1|-1}(\xi) + \operatorname{sign}(2C_b - 1)G_{|2C_b-1|-1}(\xi) - G_0(\xi). \end{aligned}$$

The occupation of the band due to the impurities is

$$\begin{aligned} N = -\frac{Q}{\pi} \{ & \operatorname{sign}(2C_a + 3)\beta(|C_a + \tfrac{3}{2}|) + \operatorname{sign}(2C_b + 3)\beta(|C_b + \tfrac{3}{2}|) \\ & + \operatorname{sign}(2C_a - 1)\beta(|C_a - \tfrac{1}{2}|) + \operatorname{sign}(2C_b - 1)\beta(|C_b - \tfrac{1}{2}|) - \beta(\tfrac{1}{2})\} \end{aligned} \tag{8}$$

which is the contribution of the impurities, where the function β is defined as $\beta(x) = \sum_{k=0}^{\infty}(-1)^k/(x + k)$. In the other case, when the band is almost empty, Q is very large and the integral equation can be similarly solved by iteration and the corresponding Wiener–Hopf-type equation can be solved straightforwardly. The leading contribution to the occupation number due to the impurities is obtained by

$$N = \frac{1}{\pi Q}\left(1 + \frac{\ln Q}{2\pi Q}\right)\left(C_a + C_b + \frac{3}{4}\right). \tag{9}$$

Similarly, we can derive the number of particles for the ferromagnetic case when the band is nearly empty or nearly full. The integration limits B and Q have been studied in detail in [35,15] for the t–J model. Using the method given by Schlottmann, we get $B = -\frac{1}{\pi} \ln\{H/[\pi R(\frac{2\pi}{e})^{1/2} - \frac{1}{2G}\pi R_G(\frac{2\pi}{e})^{1/2}]\}$ with $R_G = \int_{-Q}^{Q} \mathrm{d}k\, X_\sigma^h(k)\mathrm{e}^{\pi k}$ and $R = 1 + \int_{-Q}^{Q} \mathrm{d}\xi'\, \sigma^h(\xi') \exp(\pi\xi')$. This shows that the host determines $B(H)$ and the impurities only renormalize the integration limit B in an irrelevant way since the $1/G$ term in the above relation can be dropped. When the band is almost half-filled we get $Q = \pi[(A + 2)/(\ln 2\sqrt{2e}) - 1]/(2\ln 2)$. Hence we have that $A > \sqrt{2e}\ln 2 - 2 \approx -0.38$. This means that the Fermi level is higher for the one-dimensional electron system with impurity. When the band is almost empty Q is very large and we get $Q = (A + 2)^{-1/2}$.

The Fermi level must be above the bottom of the band. This is the same as the result given by Schlottmann [35,15]. By taking into account expression (4), we know that the susceptibility of the system with the impurities also has logarithmic corrections similar to the Heisenberg antiferromagnet [44] and the t–J model [35] without impurity.

4. Excitation and bound states

Using the string hypotheses [35]: $\lambda_\alpha^{n,j} = \lambda_\alpha^n + (n+1-2j)\frac{\mathrm{i}}{2} + \mathrm{O}(\exp(-\delta G)), n = 1, 2, \ldots, \infty$, which denotes M_n strings of complex spin rapidities of length n, corresponding to the bound spin states in spin sectors, and $q_\beta^{1,2} = \pm\lambda_\beta \pm \frac{\mathrm{i}}{2} + \mathrm{O}(\exp(-\delta G))$, corresponding to the bound or paired electron states in charge sectors, we get the following integral equations:

$$\begin{aligned} &a(\lambda, 1) + \frac{1}{2G}\sigma^G(\lambda) = \sigma(\lambda) + \sigma^h(\lambda) + [2]\sigma(\lambda) + [1]\rho(\lambda) \\ &a\left(\lambda, \frac{1}{2}\right) + \frac{1}{2G}\rho^G(\lambda) = \rho(\lambda) + \rho^h(\lambda) + [1]\sigma(\lambda) + \sum_{n=1}^{\infty}[n]\sigma_n(\lambda) \\ &[n]\rho(\lambda) + \frac{1}{2G}\sigma_n^G(\lambda) = \sigma_n^h(\lambda) + \sum_{m=1}^{\infty} A_{nm}\sigma_m(\lambda)] \end{aligned} \tag{10}$$

where σ^G and ρ^G are expressed by (3), and σ_n is the distribution density function of M_n strings of complex spin rapidities of length n. The function $\sigma_n^G(\lambda)$ is denoted by

$$\begin{aligned} \sigma_n^G(\lambda) = a\left(\lambda, \frac{n}{2}\right) + a\left(\lambda, \frac{n-2C_a}{2}\right) + a\left(\lambda, \frac{n+2C_a}{2}\right) \\ + a\left(\lambda, \frac{n-2C_b}{2}\right) + a\left(\lambda, \frac{n+2C_b}{2}\right). \end{aligned}$$

The operator A_{nm} has the expression

$$A_{nm} \equiv [|n-m|] + 2[|n-m|+2] + 2[|n-m|+4] + \cdots + 2[n+m-2] + [n+m].$$

By Fourier transforming the above integral equations we have

$$\begin{aligned} &\tilde{\sigma}_{n-1}^h(\omega) + \tilde{\sigma}_{n+1}^h(\omega) = 2\cosh\frac{\omega}{2}(\tilde{\sigma}_n(\omega) + \tilde{\sigma}_n^h(\omega)) \\ &\tilde{\rho}(\omega) + \tilde{\sigma}_2^h(\omega) + \frac{1}{2G}\exp\left(\frac{|\omega|}{2}\right)\tilde{\sigma}_1^G(\omega) = 2\cosh\frac{\omega}{2}(\tilde{\sigma}_1(\omega) + \tilde{\sigma}^h(\omega)) \\ &\tilde{\sigma}^h(\omega) + \tilde{\sigma}_1^h(\omega) + 1 = 2\cosh\frac{\omega}{2}(\tilde{\rho}(\omega) + \tilde{\rho}^h(\omega)) + \frac{1}{2G}\exp\left(\frac{|\omega|}{2}\right)\tilde{\sigma}_2^G(\omega) \\ &\exp\left(-\frac{|\omega|}{2}\right)\tilde{\sigma}^h(\omega) - \tilde{\rho}(\omega) + \exp\left(-\frac{|\omega|}{2}\right) + \frac{1}{2G}\exp\left(\frac{|\omega|}{2}\right)\tilde{\sigma}^G(\omega) \\ &\qquad = 2\cosh\frac{\omega}{2}(\tilde{\sigma}(\omega) + \tilde{\sigma}^h(\omega)) \end{aligned} \tag{11}$$

where the wave denotes a Fourier transform. The terms with the factor $1/(2G)$ correspond to the effects of the magnetic impurities in the thermodynamical limits. These equations are similar to the one-dimensional fermion gas [45] and the Anderson impurity [46]. They are only different in the correction terms from the integrable narrow-band model [35] related to the heavy-fermion systems. The distribution functions are determined by the minimization of the thermodynamic potential. The pressure, $P = -\Omega/G$, of the system has the form:

$$\begin{aligned} P = T[1]\ln(1+\zeta^{-1}) + T[2]\ln(1+\eta^{-1}) \\ + \frac{T}{2G}\left\{ \int \mathrm{d}k\, \rho^G(k)\ln(1+\zeta^{-1}(k)) + \int \mathrm{d}k\, \sigma^G(k)\ln(1+\eta^{-1}(k)) \right. \end{aligned}$$

$$+\sum_{n=1}^{\infty}\int \mathrm{d}k\,\sigma_n^G(k)\ln(1+\eta_n^{-1}(k))\Bigg\}. \qquad (12)$$

The term with factor $1/(2G)$ gives the finite-size correction of the pressure due to the impurities. By following the notations in [35], we set that $\zeta = \exp(\varepsilon/T)$, $\eta = \exp(\Psi/T)$ and $\eta_n = \exp(\varphi_n/T)$. Then we obtain

$$\begin{aligned}\rho &= -\frac{1}{2\pi}\frac{\partial\varepsilon}{\partial\chi^s}\frac{1}{1+\zeta}+\frac{1}{2G}X_\rho, \sigma_n^h = \frac{1}{2\pi}\frac{\partial\varphi_n}{\partial\chi^s}\frac{1}{1+\eta_n^{-1}}+\frac{1}{2G}X_{\sigma_n}^h\\ \sigma^h &= -\frac{1}{2\pi}\frac{\partial\Psi}{\partial\chi^s}\frac{1}{1+\eta^{-1}}+\frac{1}{2G}X_\sigma^h\end{aligned} \qquad (13)$$

and similar expressions for the complementary functions. X_ρ, X_σ^h and $X_{\sigma_n}^h$ are the finite-size corrections which satisfy the following relations:

$$\begin{aligned}&X_{\sigma_n}^h + \sum_{m=1}^{\infty} A_{nm}\eta_m^{-1}X_{\sigma_m}^h = [n]X_\rho + \sigma_n^G\\ &(1+\zeta)X_\rho + [1]\eta^{-1}X_\sigma^h + \sum_{n=1}^{\infty}[n]\eta_n^{-1}X_{\sigma_n}^h = \rho^G\\ &(1+\eta^{-1}+[2]\eta^{-1})X_\sigma^h + [1]X_\rho = \sigma^G.\end{aligned} \qquad (14)$$

The finite-size correction of the free energy due to the impurities is

$$\begin{aligned}F_i = -\frac{T}{2}\Bigg\{&\int \mathrm{d}k\,\rho^G(k)\ln(1+\zeta^{-1}(k)) + \int \mathrm{d}k\,\sigma^G(k)\ln(1+\eta^{-1}(k))\\ &+\sum_{n=1}^{\infty}\int \mathrm{d}k\,\sigma_n^G(k)\ln(1+\eta_n^{-1}(k))\Bigg\}.\end{aligned} \qquad (15)$$

When $T \to \infty$, the finite-size corrections of the free energy due to the two impurities are

$$\begin{aligned}F_i &= \frac{T}{2}\{\ln(\zeta^{\frac{9}{2}}(1+\zeta)^{\frac{1}{2}}) - \ln(\eta(1+\eta)^{\frac{3}{2}})\} \qquad C_a = C_b \to 0\\ F_i &= \frac{T}{2}\{\ln(\zeta(1+\zeta)^{\frac{1}{2}}) - \ln(\eta(1+\eta)^{-\frac{1}{2}})\} \qquad C_a = C_b \to \infty\\ F_i &= \frac{T}{2}\{\ln(\zeta^{\frac{5}{2}}(1+\zeta)^{\frac{1}{2}}) - \ln(\eta(1+\eta)^{\frac{1}{2}})\} \qquad C_a = C_b^{-1} \to 0 \text{ or } \infty\end{aligned} \qquad (16)$$

for some special limits of the coupling constants. This means that these special limits are impurity-model dependent. At the low-temperature limit, the finite-size corrections corresponding to the distribution density functions are described by

$$\begin{aligned}&X_\rho = ([0]+[2])\rho^G - [1]\sigma^G \qquad X_{\sigma_n}^h = 0 \qquad n > 0\\ &\eta^{-1}X_\sigma^h = \sigma^G - [1]\rho^G.\end{aligned} \qquad (17)$$

The free energy of the bulk is the same as the periodic t–J model. And the impurities located at the edges of the system give the finite-size correction of the free energy. From the asymptotic property of the function η_n we know that [47, 48] the impurities contribute a nonlinear term to the specific heat at the low-temperature limit. This nonlinear relation is impurity-model dependent although it is difficult to get the exact expression. The impurities also introduce bound states in the present system. When $-\frac{3}{2} < C_{a,b} < -1$, the bound state can be formed by

$$\begin{aligned}&q_0 = \mathrm{i}C_{a,b} + \mathrm{i} \qquad \lambda_0 = -\frac{\mathrm{i}}{3}C_{a,b} - \frac{\mathrm{i}}{3}\\ &q_+ = -\frac{\mathrm{i}}{3}C_{a,b} + \frac{\mathrm{i}}{6} \qquad q_- = -\frac{\mathrm{i}}{3}C_{a,b} - \frac{5\mathrm{i}}{6}.\end{aligned} \qquad (18)$$

This bound state carries the energy

$$E_{a,b}^{bou} = -\frac{2(38C_{a,b}^2 + 76C_{a,b} + 11)}{(4C_{a,b}^2 + 8C_{a,b} + 3)(C_{a,b}^2 + 2C_{a,b} - 8)}. \tag{19}$$

The moment of the centre of the bound state is zero. In this case, the Kondo couplings between the bulk and the impurities are antiferromagnetic with the repulsive impurity potentials. When the parameters $C_{a,b}$ fall into the regime $-\frac{5}{2} < C_{a,b} < -\frac{3}{2}$, the system also has the bound state formed by the four imaginary modes (18) and is localized at the ends of the system. It carries the energy (19) and the moment of the centre is zero, in which the Kondo couplings are ferromagnetic and the impurity potentials are attractive. Obviously, by making the transformations $q_0 \to -q_0$, $\lambda_0 \to -\lambda_0$ and $q_\pm \to -q_\pm$ in relations (18), we also get the boundary bound states with $-\frac{5}{2} < C_{a,b} < -1$. The moment of the centre of the boundary bound state is zero and it also carries the energy (19). Notice that the above boundary bound states introduced by the impurities are different from the ones related to the $SU(3)$ invariant t–J model [31, 32].

5. Concluding remarks

In this paper, with respect to the investigations in [29, 31, 32], we have studied the impurity model related to the traditional t–J model which has graded FFB super-algebra. The finite-size correction of the low-field magnetization due to the magnetic impurities is obtained. The charge fluctuation of the system is discussed for the singlet state scattering and the thermodynamical Bethe ansatz equations for the excited state are obtained for the impurity model. The contribution of magnetic impurities to the free energy is derived in the thermodynamical limit. The properties of the system in the low- and high-temperature limits are discussed. The relation between the external field and the integral limit of the system is obtained and we find that the Fermi level is higher for the one-dimensional electron system with impurity when the band is almost half-filled. We have also found that the system has the boundary bound states introduced by the impurities and formed by four imaginary modes of the rapidities, which are different from the ones in the impurity model within the framework of the $SU(3)$ invariant t–J model.

The impurity part of the Hamiltonian in our model has a simple and compact form since the scattering matrices in the bulk of the system are the tangent (or cotangent) functions of the half moments of the electrons and the boundary scattering matrices between the impurities and the electrons can be factorized as two terms which are similar to the R matrix of the system. The kinetic energy in our model is dependent on the electron momentum and the impurities make no contribution to the kinetic energy. This situation in the present model coincides with the features of the Kondo problem in [1]. The interaction parameters of the impurities with conduction-band electrons run from the ferromagnetic case to the antiferromagnetic case. This is an exactly solved model in one dimension dealing with the strongly correlated electron system with impurities and its kinetic energy is nonlinear in the electron momentum. In the Kondo problem [49] one studies the low-temperature behaviour of a system with magnetic impurities represented by localized spins that couple to the conduction electrons via a spin exchange interaction. The kinetic energy of the system is linear in the conduction-band electrons, which means that only electrons are very close to the Fermi surface.

In the magnetic impurity model (1) the impurities are coupled to the two ends of the electron system. The integrability condition of the system imposes restrictions on the interaction parameters between the nearest-neighbour sites of electrons and the interaction parameters between electrons and impurities so that they take the special forms. Since in heavy-fermion systems the case of singlet scattering is important, we deal with this case, in detail, on the contributions of the magnetic impurities. The finite-size corrections due to the magnetic impurities have the factor $1/(2G)$, which is concordant with [50]. The low-field magnetization of the system is derived and it means that the susceptibility also has logarithmic correction when the impurities exist. The charge fluctuation of the system is discussed for the singlet state scattering and the explicit expression of free energy due to the magnetic impurities is given under the limits of the interaction with conduction electrons. When the Kondo coupling falls into the antiferromagnetic region and the impurity potential is repulsive, or the impurity potential is attractive but the Kondo coupling falls into the ferromagnetic region, the system has boundary bound states composed by four imaginary modes of the rapidities in the spin and charge sectors. The amplitudes of these imaginary modes are different, which means the system has a nonzero wavefunction. The boundary bound states carrying the energy and moments of the centres are zero. Notice that the main investigations in this paper concern thermodynamic limits and the Bethe ansatz equations have been written as coupled integral equations. It is worthwile studying the impurity model further for the finite-lattice system related to the t–J model with the singlet states scattering. In fact, a method to calculate the leading-order finite-size corrections to the ground state energy has been given by de Vega and Woynarovich in [51]. Some other integrable models have also been studied using finite-size scaling techniques [52–57]. It would also be interesting to study the present impurity model further in the different sectors and the critical property contributed by the magnetic impurities. Finally, we point out that the boundary bound state formed by the four imaginary modes of the rapidities is obtained for our impurity model. We can ask the question: 'Are there other bound states introduced by the impurities in the model?'. It would prove a challenging problem to find them and to consider the effects (such as magnetization, band-occupation and corresponding energy, etc) of these bound states in the ground state of the system. We wish this question to remain as an open problem for further investigations.

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